Multiphase Flow and Heat Transfer

ME546

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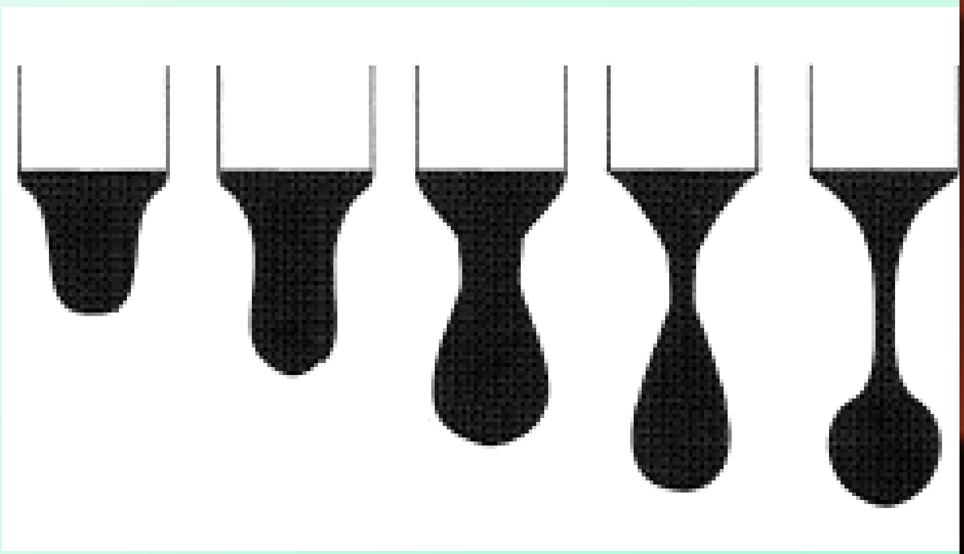
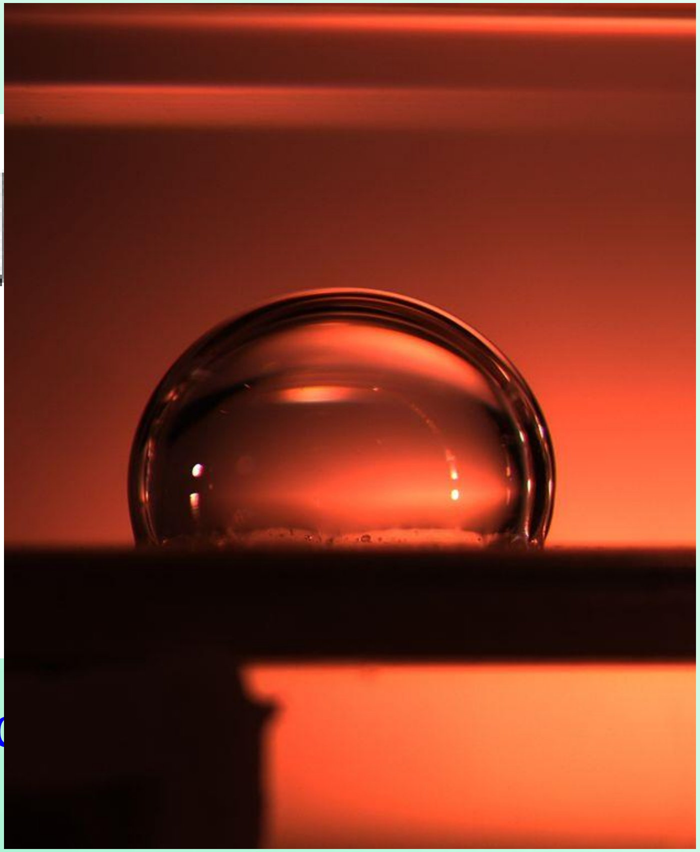
Pendant and Sessile Drops



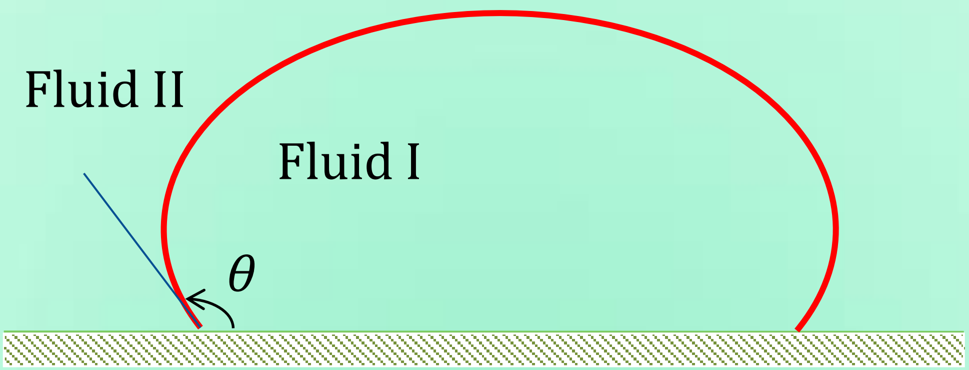
Pendant droplet detachment sequenc



Sessile water droplet immersed in
oil and resting on a brass surface

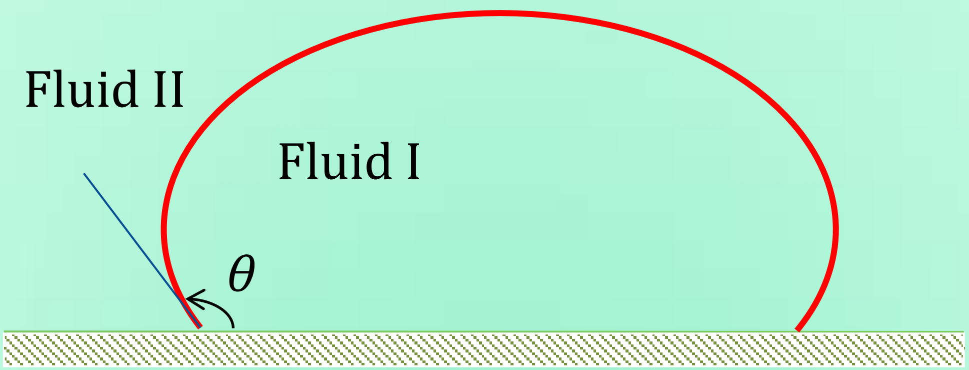
Interface Shape at Equilibrium

- Consider a sessile drop sitting on a smooth solid surface inside another fluid.



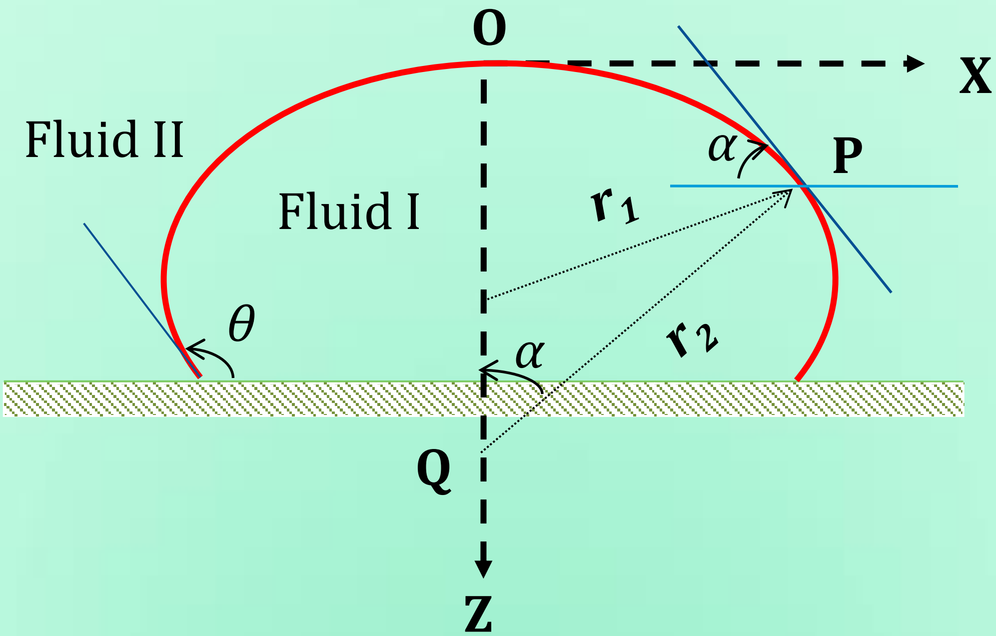
Interface Shape at Equilibrium

- In the absence of gravity, the drop takes spherical shape with least surface area.
- The drop is deformed by gravity. The center of mass of the drop is forced to be lowered by gravity.
- This increases the surface area, which is opposed by surface tension force.
- Assume there are no external forces acting on the drop.



Interface Shape at Equilibrium

- Origin of the coordinate system is O, located at the apex of the surface. The drop is assumed to be axisymmetric.
- At O: Radii of curvature: $r_1 = r_2 = r_0$
- At any P: Radii of curvature are r_1 and r_2

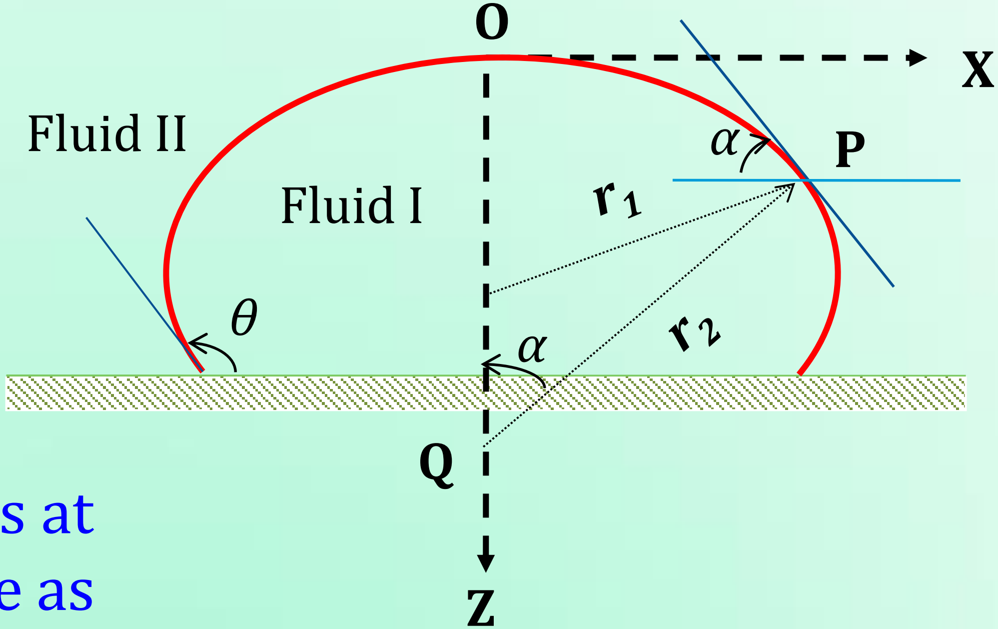


At origin using Young-Laplace:

$$\left(P_I - P_{II}\right)_0 = \frac{2\sigma}{r_0}$$

At point P:

$$\left(P_I - P_{II}\right)_p = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$



Hydrostatic pressure heads at any point (P) on interface as seen from fluids I and II:

$$P_{Ip} = P_{I0} + \rho_I g z$$

$$P_{IIp} = P_{II0} + \rho_{II} g z$$

$$r_2 = \frac{x}{\sin\alpha}$$

Bashforth-Adams Equation

$$\sigma\left(\frac{1}{r_1}+\frac{\sin\alpha}{x}\right)=\frac{2\sigma}{r_0}+\left(\rho_I-\rho_{II}\right)gz$$

$$\frac{1}{\left(r_1/r_0\right)}+\frac{\sin\alpha}{\left(x/r_0\right)}=2+\left[\frac{\left(\rho_I-\rho_{II}\right)gr_0^2}{\sigma}\right]\frac{z}{r_0}$$

Bashforth-Adams Equation

$$\boxed{\frac{1}{\left(r_1/r_0\right)}+\frac{\sin\alpha}{\left(x/r_0\right)}=2+\mathrm{Bo}\,\frac{z}{r_0}}$$

$$\mathrm{Bo}=\frac{\left(\rho_I-\rho_{II}\right)gr_0^2}{\sigma}$$

Bashforth-Adams Equation

$$\frac{1}{(r_1/r_0)}+\frac{\sin\alpha}{(x/r_0)}=2+\mathrm{Bo}\frac{z}{r_0}$$

$$\mathrm{Bo}=\frac{(\rho_I-\rho_{II})gr_0^2}{\sigma}$$

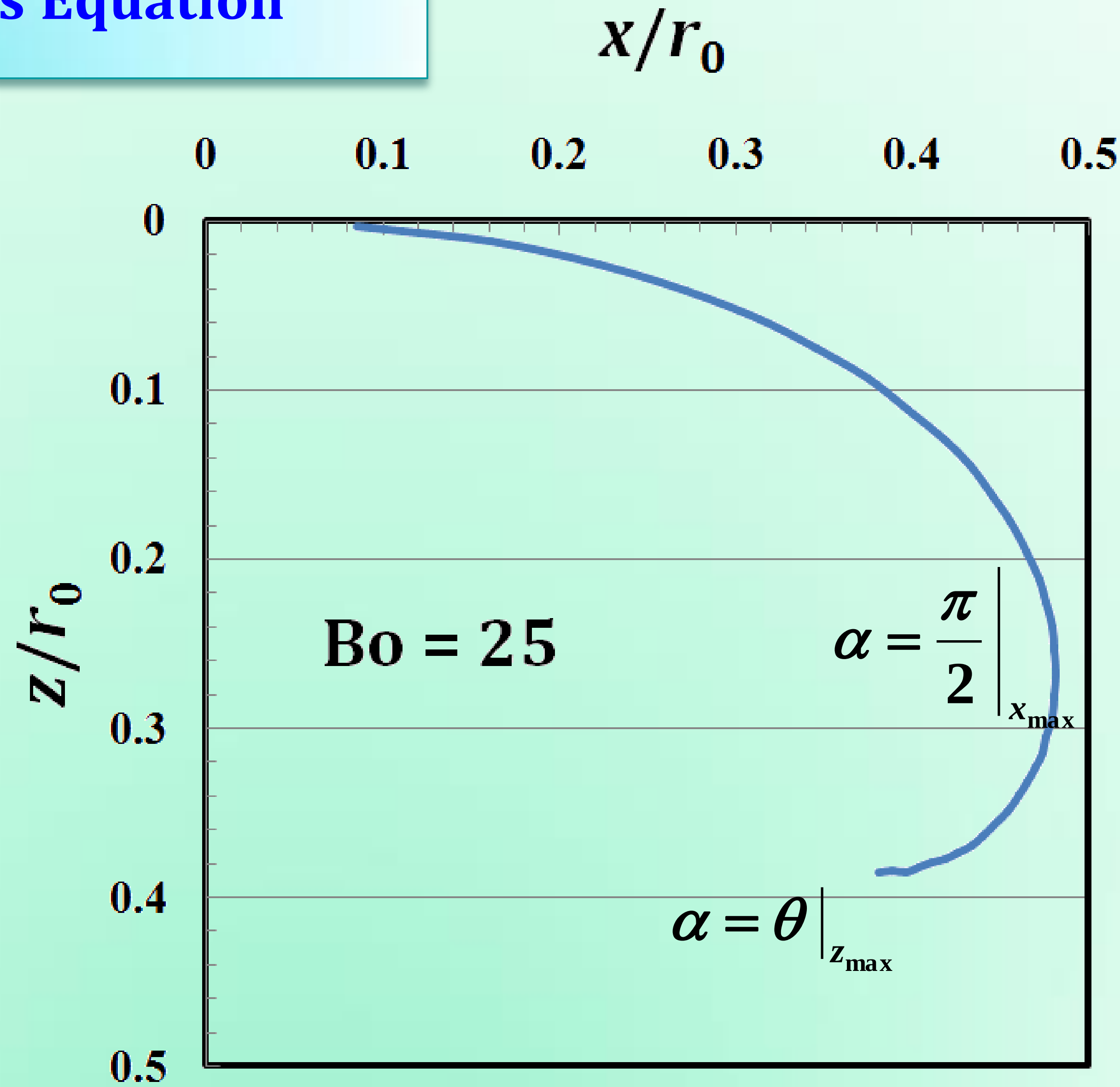
$r_1 = f\,(dz/dx,\, d^2z/dx^2)$

Numerically solvable with appropriate BCs.

Solution in tabular form (Bashforth and Adams in 1883)

- Variation of z/r_0 and x/r_0 with α at a given Bo

Bashforth-Adams Equation



Profile of drop predicted by Bashforth-Adams equation at Bo = 25

Bond Number

$$\mathbf{Bo} = \frac{(\rho_I - \rho_{II}) g r_0^2}{\sigma}$$

Ratio of the gravity force to the force due to surface tension.

Bo << 1 Drop will not deform significantly

Bo >> 1 Large deformation of the drop

Bo << 1, if

the drop is small

or the interfacial tension is large

or the density difference between the two liquid is low

$$\mathbf{Bo} = \frac{(\rho_I - \rho_{II}) g r_0^2}{\sigma}$$

$\rho_I > \rho_{II}$

Bo > 0, drop shape is oblate

Weight of the drop flattens the surface

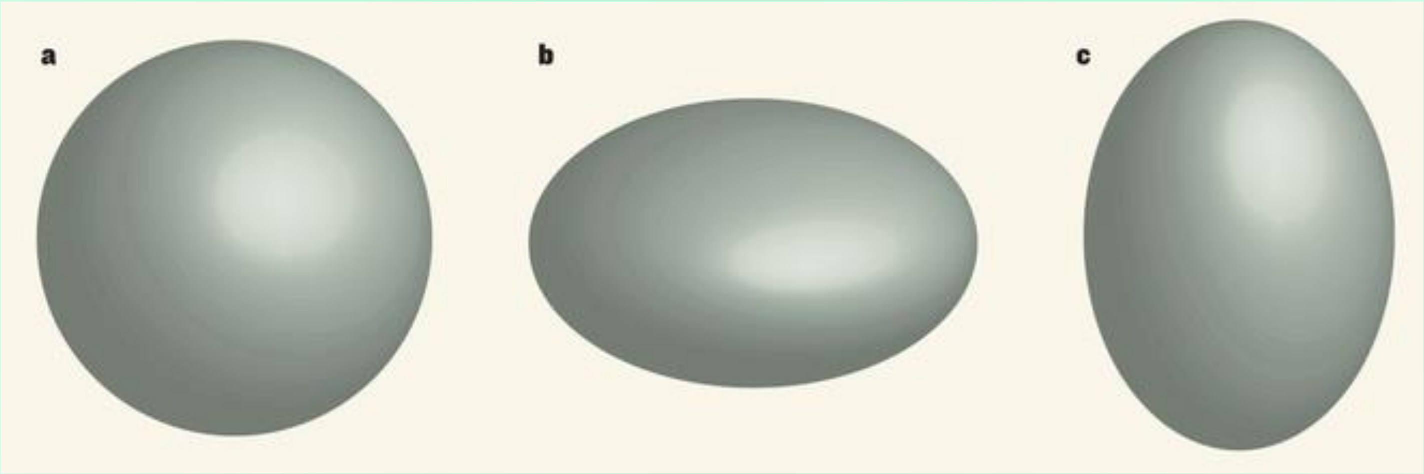
Eg: Rain drop, drop on a surface

$\rho_I < \rho_{II}$

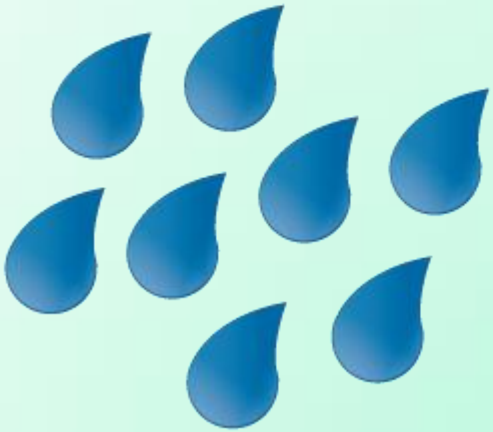
Bo < 0, drop shape is prolate

Buoyancy elongates shape vertically

Eg: Vapor bubble in liquid



Shape of Raindrops



$$\mathbf{Bo} = \frac{(\rho_I - \rho_{II}) g r_0^2}{\sigma}$$

$$L_c = \sqrt{\frac{\sigma}{(\rho_l - \rho_v) g}}$$

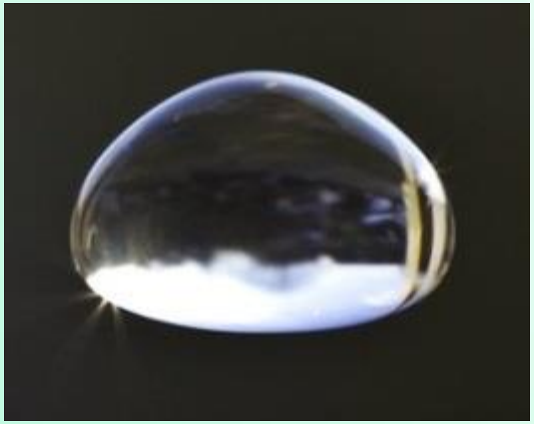
$$\mathbf{Bo} = \left(\frac{r_0}{L_c}\right)^2$$

$r_0 << L_c \quad \Rightarrow \quad$ Gravity is negligible

$L_c \simeq$? mm, air-water interface at 25°C

Shape of Raindrops

$$\mathbf{Bo} = \left(\frac{r_0}{L_c} \right)^2$$



$L_c \simeq 2.7$ mm, air-water interface at 25°C

$r_0 \simeq$ 2 mm, Nearly perfect sphere

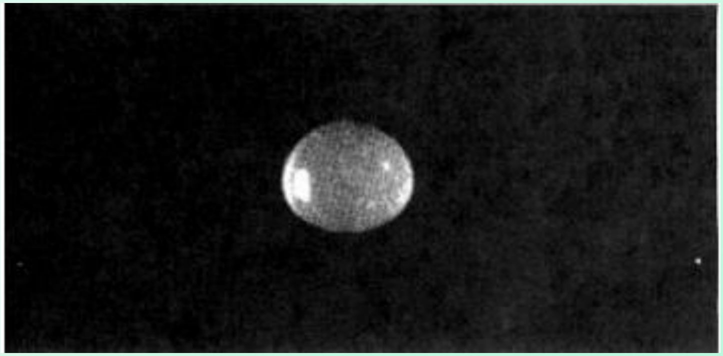
$r_0 >> 2$ mm, increasingly flattened

$r_0 > 4.5$ mm, Raindrop breaks into smaller drops due to interaction with air.

Falling raindrops are deformed by the interaction with the air and never take the familiar teardrop shape with a pointed tail and a rounded bottom head.

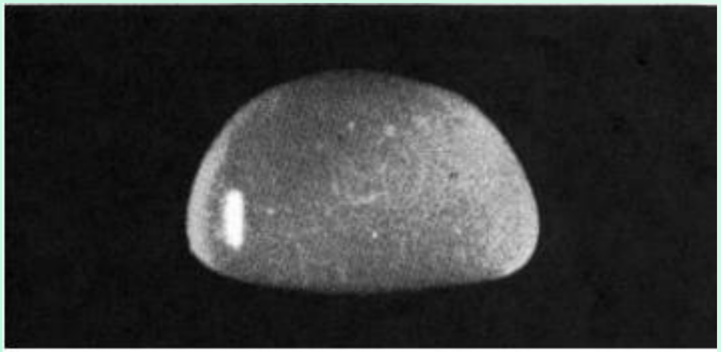
Shape of Raindrops

Steadily falling raindrops are subject to the combined effects of surface tension, gravity, friction, and air currents.



Small raindrops are dominated by surface tension (nearly spherical)

Here r_0 is slightly greater than L_c (2.7 mm) and the drop is slightly oval



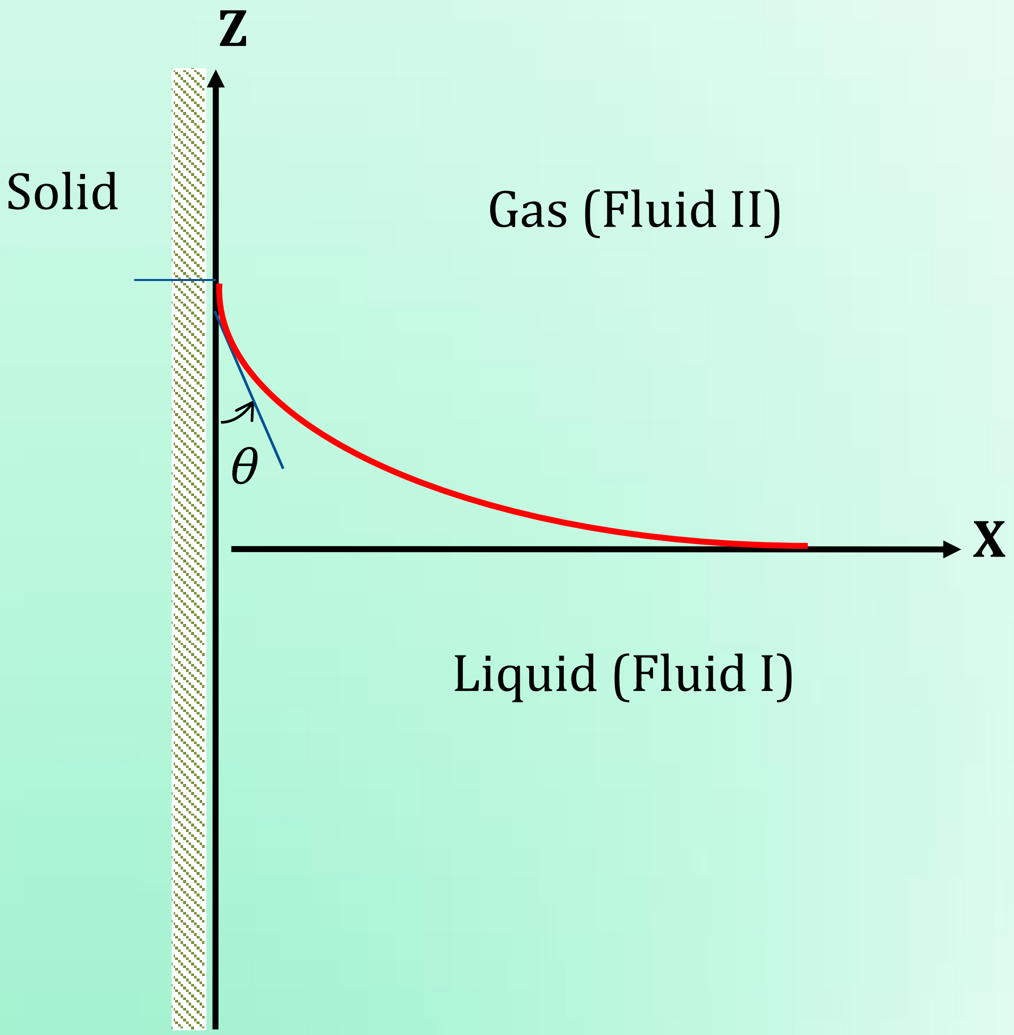
Larger raindrops assume a typical "hamburger" shaper

Liquid Climbing the wall

Shape of a free liquid surface meeting a plane vertical wall.

If the liquid wets the wall ($\theta < 90°$), the liquid level will rise as the wall is approached, meeting the wall at θ.

Consider a 2D configuration:



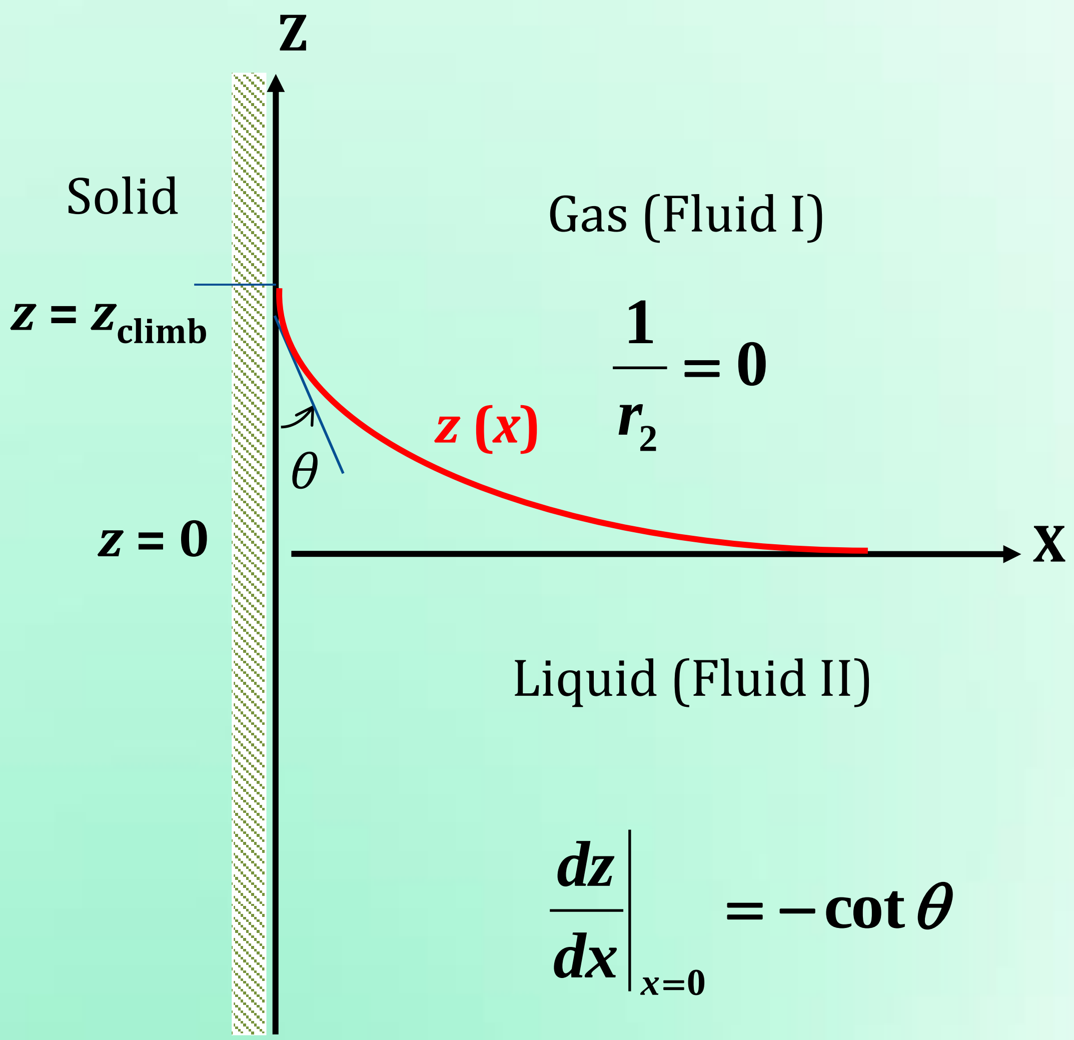
Liquid Climbing the wall

$$P_I - P_{II} = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

$$P_I = P_0 - \rho_v g z$$

$$P_{II} = P_0 - \rho_l g z$$

$$\frac{(\rho_l - \rho_v) g z}{\sigma} - \frac{1}{r_1} = 0$$



$$\frac{1}{r_2} = 0$$

$$\left. \frac{dz}{dx} \right|_{x=0} = -\cot\theta$$

Radius of Curvature

$$(x-a)^2+(z-b)^2=R_c^2$$

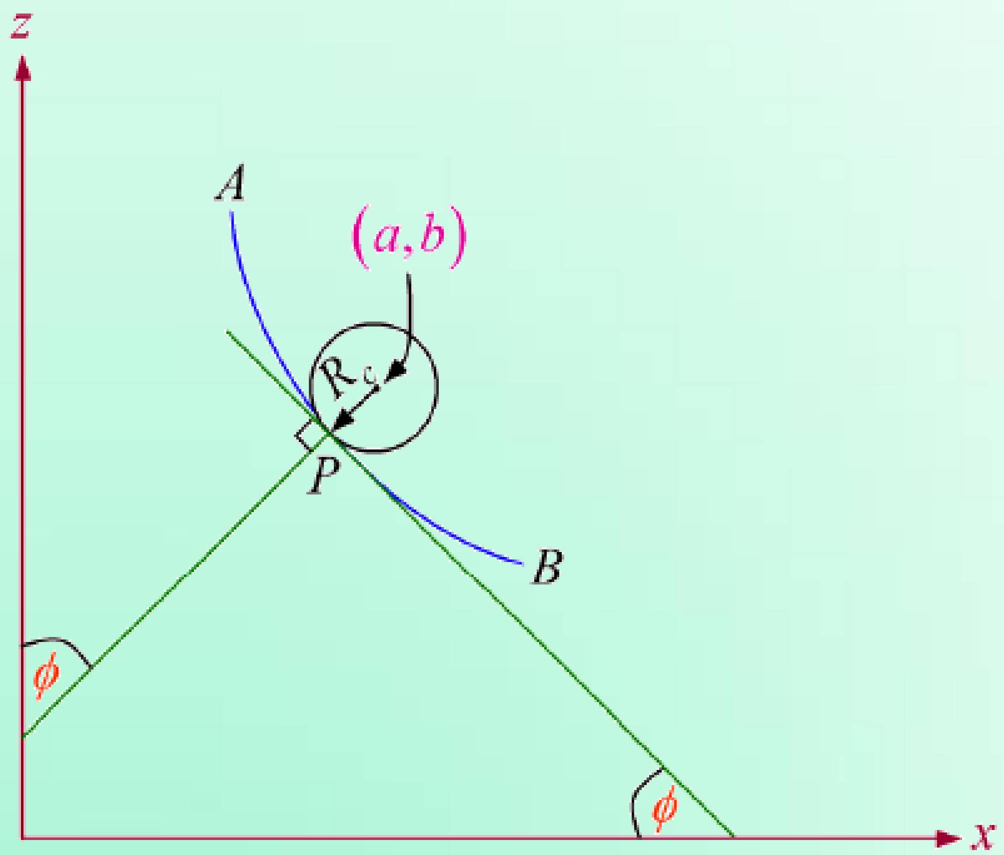
Differentiating w.r.to *x*

$$\frac{dz}{dx}=-\frac{x-a}{z-b}$$

However

$$\tan\phi=-\frac{dz}{dx}$$

$$\tan\phi=\frac{x-a}{z-b}$$



Radius of Curvature

$$\cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}} = \pm\frac{z-b}{R_c}$$

$$\frac{d\cos\phi}{dz} = \pm\frac{1}{R_c}$$

$$\frac{d\cos\phi}{dz} = \frac{d}{dz}\left[1+\left(\frac{dz}{dx}\right)^2\right]^{-\frac{1}{2}} = \frac{dx}{dz}\frac{d}{dx}\left[1+\left(\frac{dz}{dx}\right)^2\right]^{-\frac{1}{2}}$$

$$\frac{1}{R_c} = \pm\frac{d^2z/dx^2}{\left[1+\left(\frac{dz}{dx}\right)^2\right]^{\frac{3}{2}}}$$

$$\frac{1}{r_1} = \frac{z''}{\left[1+(z')^2\right]^{\frac{3}{2}}} \qquad z' = \frac{dz}{dx} \qquad z'' = \frac{d^2 z}{dx^2}$$

$$\frac{(\rho_l - \rho_v) g z}{\sigma} - \frac{1}{r_1} = 0$$

$$\frac{(\rho_l - \rho_v) g z}{\sigma} - \left[1+(z')^2\right]^{-\frac{3}{2}} z'' = 0$$

Multiplying by z'

$$\frac{(\rho_l - \rho_v) g}{\sigma} z(z') - \frac{1}{2}\left[1+(z')^2\right]^{-\frac{3}{2}} (2z'z'') = 0$$

Integrating

$$\frac{(\rho_l - \rho_v)g}{\sigma} \int z(z') - \frac{1}{2} \int \left[1 + (z')^2\right]^{-\frac{3}{2}} (2z'z'') = 0$$

Since, $$-\frac{1}{2}\left[1+(z')^2\right]^{-\frac{3}{2}}(2z'z'') = \frac{d}{dx}\left\{\left[1+(z')^2\right]^{-\frac{1}{2}}\right\}$$

$$\frac{(\rho_l - \rho_v)g}{2\sigma} z^2 + \left[1+(z')^2\right]^{-\frac{1}{2}} = \mathbf{C}$$

BCs: $z, z' = 0$ as $x \to \infty \Rightarrow$ Integral constant, $C = 1$

Liquid Climbing the wall

$$\frac{(\rho_l - \rho_v)g}{2\sigma} z^2 + \left[1 + (z')^2\right]^{-\frac{1}{2}} = 1$$

$$z'\big|_{x=0} = -\cot\theta$$

The height to which the liquid climbs at the vertical wall

$$z\big|_{x=0} = z_{\text{climb}} = \left[\frac{2\sigma(1 - \sin\theta)}{(\rho_l - \rho_v)g}\right]^{\frac{1}{2}}$$

Liquid Climbing the wall

The shape of the interface:

$$\frac{x}{L_c} = \cosh^{-1}\left(\frac{2L_c}{z}\right) - \cosh^{-1}\left(\frac{2L_c}{z_{\text{climb}}}\right) - \left(4 + \frac{z^2}{L_c^2}\right)^{\frac{1}{2}} + \left(4 + \frac{z_{\text{climb}}^2}{L_c^2}\right)^{\frac{1}{2}}$$

$$L_c = \left[\frac{\sigma}{(\rho_l - \rho_v)g}\right]^{\frac{1}{2}}$$

Liquid Climbing the wall

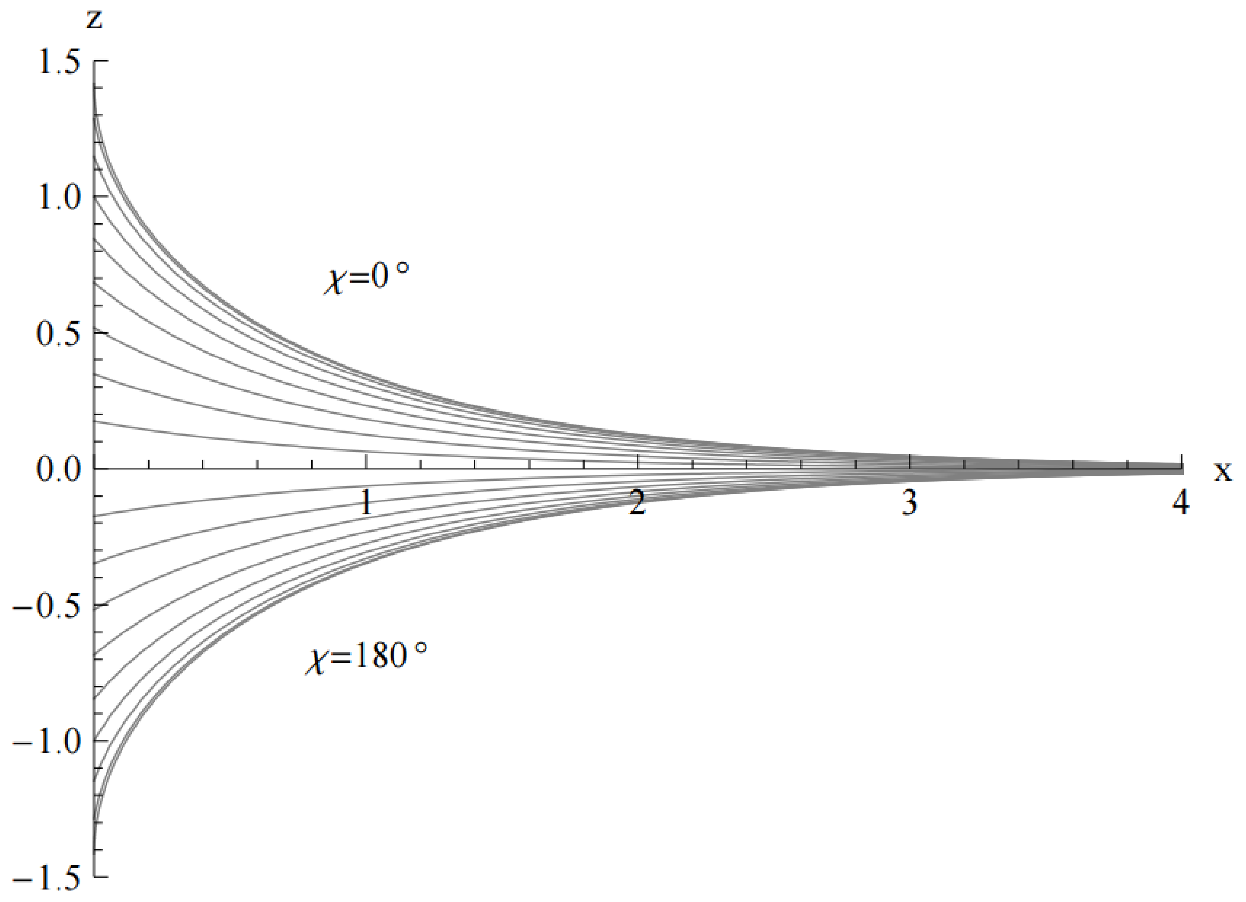


Figure 5.7. Flat-wall meniscus (in units of $L_c = 1$) for all contact angles in steps of 10°. Close to the wall the menisci are linear and far from the wall they approach the x-axis exponentially.

Capillary Rise or Depression

A cylindrical container is filled with saturated liquid R-134a and its vapor at 32°C. Determine the height to which the liquid will climb the vertical walls of the container if the contact angle with the walls is 5°.

At 32°C, σ = 0.0072 N/m, ρ_v = 39.8 kg/m^3, ρ_l = 1180 kg/m^3.

$$z_{\text{climb}} = \left[\frac{2\sigma(1-\sin\theta)}{(\rho_l - \rho_v)g} \right]^{\frac{1}{2}}$$

$$z_{\text{climb}} = 1.1\,\text{mm}$$

Marangoni Forces

Any variation in surface tension along an interface will create tangential (shear) forces, known as **Marangoni Forces**.

This variation can arise from inhomogeneous material properties, or from temperature variations.

Unless balanced by other forces, these shear forces cannot be sustained in a liquid at rest - will set it into motion.